impossible to tune the filter due to perturbation of the coupling values by the coupling and tuning screws.

REFERENCES

[1] H. A. Bethe, "Theory of side windows in waveguides," M.I.T. Radiation Lab., Cambridge, Mass., Rep. 43-27, Apr. 1943.
[2] S. B. Cohn, "Microwave coupling by large apertures," *Proc. IRE*, vol. 40, pp. 696–699, June 1952.
[3] A. E. Atia and A. E Williams, "Nonminimum-phase optimum-amplitude bandpass waveguide filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 425–431, Apr. 1974.
[4] A. E. Atia, A. E. Williams, and R. W. Newcomb, "Narrow-band multiple-coupled cavity synthesis," *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 649–655, Sept. 1974.

Loss Considerations for Microstrip Resonators

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Abstract—**The influence of radiation losses on the Q of microstrip resonators is shown for a variety of frequencies, characteristic impedances, substrate materials, and thicknesses. Radiation becomes a dominant factor at higher frequencies, especially for low-impedance lines and thick substrates with a low dielectric constant.**

I. INTRODUCTION

The designer of microstrip circuits is often faced with the question of what the best substrate thickness is, and what characteristic impedance should be used if low loss is of prime importance. The particular circuit may be a high-Q resonator, a coupled-line filter, or some general matching network. The losses encountered with microstrip circuits divide into conduction, dielectric, and radiation losses. The first two loss factors have been dealt with extensively in the literature and good approximations exist [1]. Radiation losses are less well understood although several articles [2]–[6] have appeared in the last few years, and quantitative solutions are difficult to come by. The most comprehensive theoretical treatment of radiation from various circuit discontinuities was given by Lewin [6] back in 1960, and has been more recently supplemented by Sobol [3], [4].

Lewin's results, applied to an open-ended microstrip line, are used in this short paper to explore the effect radiation has on the overall Q of microstrip resonators for different values of characteristic impedance, frequency, dielectric constant, and substrate thickness. We will also show that by correcting an error in the definition of the fractional radiated power in previous papers [2], [5], the radiation losses based on Lewin's derivation are indeed in reasonably good agreement with experimental results [2] obtained earlier.

II. LOSS CALCULATIONS

Many microstrip circuits include open-circuited matching stubs and $\lambda_g/4$ or $\lambda_g/2$ resonators which have a tendency to radiate substantial amounts of RF power under certain conditions. This radiated power is either lost to the outside in open structures, or may lead to unwanted cross coupling between various circuit elements within a closed housing. Sometimes, lossy damping material is employed to absorb the radiated power and reduce cross coupling in enclosed structures. For the circuit designer, it would thus be very helpful to know in advance what losses he can expect as a function of prime design parameters such as characteristic impendance Z_0, dielectric constant ϵ_r, frequency f, and substrate thickness h.

The various loss contributions for a microstrip resonator can be represented by Q values in the form

$$Q = \frac{2\pi f_0 U}{W} \tag{1}$$

where f_0 is the resonance frequency, U is the stored energy, and W is the average power lost for a $\lambda_g/4$ microstrip resonator as shown in Fig. 1. The overall Q_t of the resonator is given by

$$\frac{1}{Q_t} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r} \tag{2}$$

where

$$Q_c = \frac{2\pi f_0 U}{W_c} = \frac{\pi}{\alpha_c \lambda_g} \quad \text{conductor losses} \tag{3}$$

$$Q_d = \frac{1}{tg\delta}\left(1 + \frac{1-q}{q\epsilon_r}\right) \quad \text{dielectric losses} \tag{4}$$

$$Q_r = \frac{2\pi f_0 U}{W_r} \quad \text{radiation losses} \tag{5}$$

where

- W_c average power lost in the conductors;
- W_r average power lost due to radiation;
- α_c conductor attenuation constant;
- $tg\delta$ dielectric loss tangent;
- q dielectric filling factor (fraction of total fields in the dielectric).

The following assumptions are made in the calculation of the individual Q values with the aim of keeping the results as close as possible to realistic conditions. Thus, dielectric losses, surface roughness, and dispersion are included, although their influence on the overall Q_t in many cases is relatively minor.

1) Dielectric losses: the calculations are based on an alumina ceramic with $\epsilon_r = 10$ and $tg\delta = 10^{-4}$; for comparison, a low dielectric constant material RT Duroid 5870 with $\epsilon_r = 2.35$ and $tg\delta = 10^{-3}$ is also included. The dielectric filling factor q varies typically from 0.5 to 1 which only in the case of low ϵ_r leads to a sizable correction for Q_d, according to (4). For alumina, the effect of q can be neglected.

2) Surface roughness: a uniform surface roughness of 5-μin rms

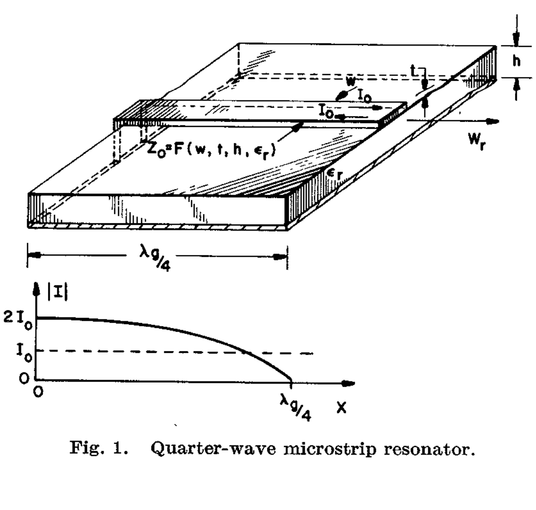

Fig. 1. Quarter-wave microstrip resonator.

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is assumed for the alumina. According to Morgan [7], this causes a reduction in Q_c by a factor of 1.04 at 8 GHz. In the case of Duroid, which has a much higher surface roughness (45-μin rms), the Q_c is reduced by a factor of 1.8 at 8 GHz, decreasing to 1.3 at 1 GHz.

3) Dispersion: A quasi-TEM mode of propagation is assumed to exist. The effective dielectric constant ϵ_{eff} starts to deviate from the static value above 1 GHz and thus changes the characteristic impedance Z_0 and the guide wavelength λ_g as a function of frequency. The dispersion is taken into account following the simplified analysis by Carlin [8].

4) Conduction losses: the corrected curves of Pucel *et al.* [1] are used with an assumed copper-conductor thickness of 0.3 mil for the alumina and 1.4 mil for the Duroid.

5) Radiation losses: the power radiated from an open-circuited microstrip line is calculated according to Lewin [6]. His derivation assumes the dielectric to end together with the resonator as shown in Fig. 1. This does not correspond to most practical cases where the dielectric substrate extends beyond the resonator, but it still provides a good approximation. The radiation contribution and cross coupling from other sources are neglected.

Due to the semiempirical nature of many of the correction factors, it is not possible to derive the various Q-parameters as a function of simple normalized quantities. Normalization is sacrificed in favor of showing the Q values as a function of relevant circuit-design parameters, thus providing a better understanding of the strong influence radiation losses have on the overall circuit Q.

The conductor losses are calculated according to

$$Q_c = \frac{\pi}{\alpha_c \lambda_g} = \frac{Z_{01} h \pi}{R_s \lambda x} \tag{6}$$

where Z_{01} is the geometric free-space characteristic impedance, h is the substrate thickness, R_s is the surface-skin resistivity, λ is the free-space wavelength, and $x = \alpha_c Z_0 h / R_s$ is the loss factor from Pucel *et al.* [1]. The relation to the characteristic line impedance Z_0 is established through

$$Z_0 = \frac{Z_{01}}{\epsilon_{eff}^{1/2}} \tag{7}$$

where ϵ_{eff} is determined by the dispersion relation after Carlin [8].

The radiation Q is calculated from (5) by substituting for U the stored energy of a $\lambda_g/4$ resonator having a current I_0 of 1 A flowing on the line as shown in Fig. 1.

$$U = \frac{Z_0}{4 f_0}. \tag{8}$$

Together with the power radiated from the open end as defined by Lewin

$$W_r = 240\,\pi^2 (h/\lambda)^2 F(\epsilon_{eff})$$

where

$$F(\epsilon_{eff}) = \frac{\epsilon_{eff} + 1}{\epsilon_{eff}} - \frac{(\epsilon_{eff} - 1)^2}{2\epsilon_{eff}^{3/2}} \ln \frac{\epsilon_{eff}^{1/2} + 1}{\epsilon_{eff}^{1/2} - 1} \tag{9}$$

the radiation Q becomes

$$Q_r = \frac{Z_0}{480\pi (h/\lambda)^2 F(\epsilon_{eff})}. \tag{10}$$

The Q values, calculated on the basis of the previous equations for a $\lambda_g/4$ resonator defined on either an alumina ceramic ($\epsilon_r = 10$) or Duroid ($\epsilon_r = 2.35$), for various substrate thicknesses and frequencies are shown in Figs. 2 and 3. To simplify the graphs and show the effect of radiation losses more clearly, the dielectric losses are combined with conductor losses in the form of a circuit quality factor

$$Q_0 = \frac{Q_c Q_d}{Q_c + Q_d}. \tag{11}$$

For the alumina substrates which confine the fields much more closely than Duroid, the radiation losses are not very pronounced at low frequencies. At higher frequencies, however, and especially for low characteristic-impedance lines and thick substrates, radiation, dominates, by far, the overall losses. Thus the commonly accepted rule for high-Q microstrip circuits, which is to use thick substrates and lines with low characteristic impedance, is self-defeating due to the high radiation losses incurred under these conditions. Although the energy may not truly be lost in a fully enclosed circuit configuration, the high radiation level will contribute greatly to extraneous cross coupling and adversely affect the overall performance of the circuit.

The other important conclusion, clearly illustrated by the graphs, is that circuits fabricated on Duroid, which is frequently used for cost reasons and/or to keep the circuit dimension large and therefore the tolerance requirements uncritical, have severe radiation-loss problems. High-Q circuits on Duroid require careful individual shielding at frequencies as low as 2 GHz.

III. COMPARISON WITH EXPERIMENTAL RESULTS

Various measurements have been reported in the past which showed relatively good agreement between theoretical and experimental results for the conductor and dielectric losses of microstrip lines. However, very little exists on actual measurements of radiation losses. A series of tests was performed some time ago [2] aimed at determining the fractional radiated power P_r/P_t of microstrip resonators as a function of h/λ and ϵ_r, but due to an error in the interpretation of P_t, the agreement with theory was rather poor and had to be corrected by an empirical coefficient.

References [2, eqs. (2) and (5)] and [5, eq. (25)] are incorrect if P_t is defined as the total power dissipated due to radiation, conductor, and dielectric losses. Lewin's derivation, on which the results of [2] and [5] are based, uses the ratio of power radiated to power *incident* on the open end of the strip line, which is not the total power dissipated. Reference [5, eq. (25)] which follows:

$$\frac{P_r}{P_T} = 240\pi^2 (h/\lambda)^2 \frac{F(\epsilon_{eff})}{Z_0}$$

is thus correct only if P_T is defined as power incident on the open-ended line. In order to obtain the true fractional radiated power from a resonator, the Q of the resonator must be taken into account.

It will be shown, as follows, that the previous measurements [2] do indeed agree well with theory if the proper definition for P_r/P_t is used. Since the measurements were performed by comparing the Q's of $n\lambda_g/2$ resonators, with and without a metal enclosure, (8)–(10) must be modified as follows to account for the electrical length of the resonator:

$$U = \frac{n Z_0}{2f} \tag{12}$$

$$W_r = 480\pi^2 (h/\lambda)^2 F(\epsilon_{eff}) \tag{13}$$

$$Q_r = \frac{n Z_0}{480\pi (h/\lambda)^2 F(\epsilon_{eff}).} \tag{14}$$

The fractional radiated power is then defined in terms of calculated values by

$$\frac{P_r}{P_t} = \frac{Q_0}{Q_0 + Q_r} \tag{15}$$

where

$$\frac{1}{Q_0} = \frac{1}{Q_c} + \frac{1}{Q_d}. \tag{16}$$

Equation (15) reduces in the limit for $Q_r \gg Q_0$ to

$$\frac{P_r}{P_t} = \frac{Q_0}{n} \frac{480\pi (h/\lambda)^2 F(\epsilon_{eff})}{Z_0} \tag{17}$$

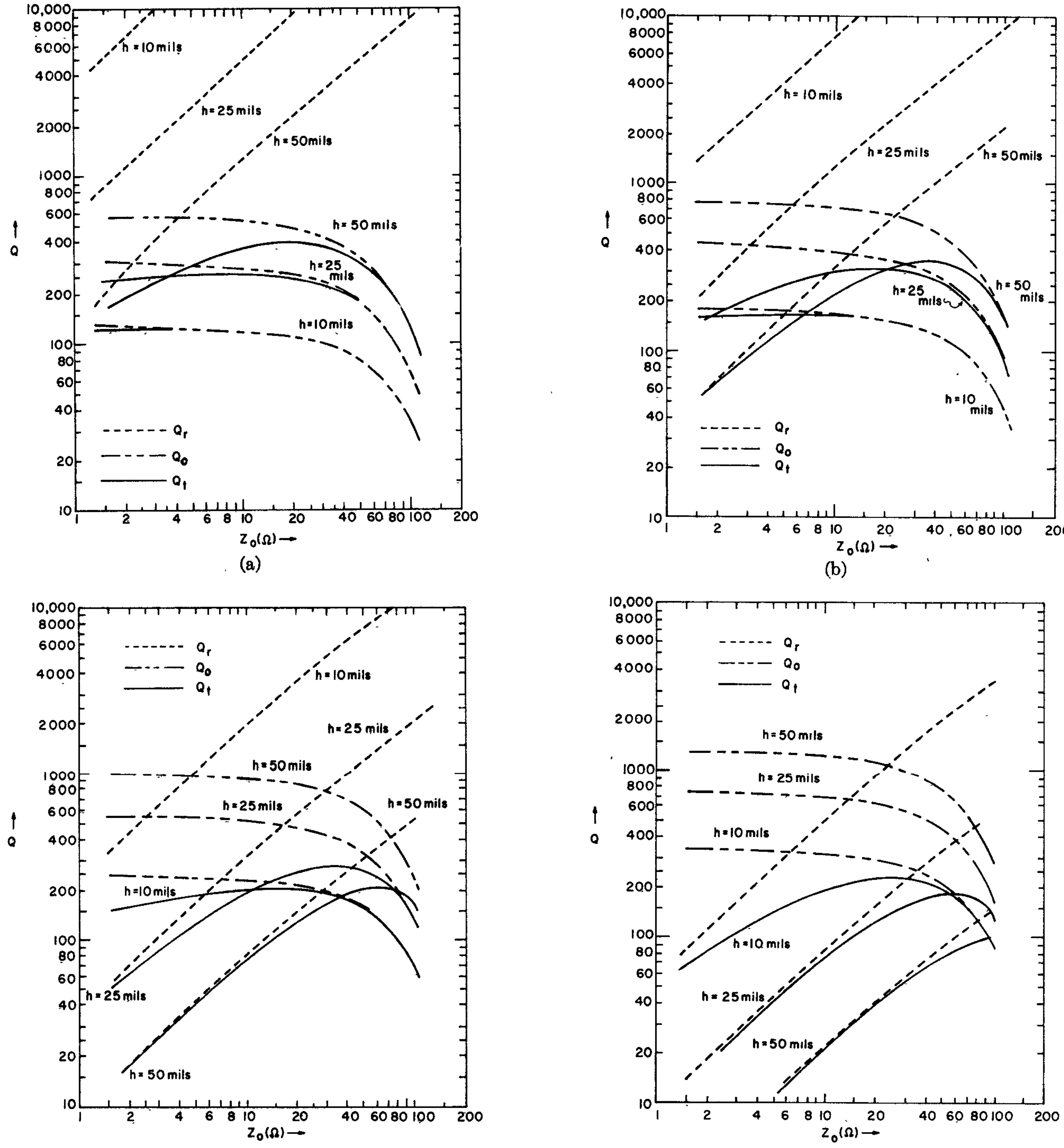

Fig. 2. Q factors for $\lambda g/4$ microstrip resonator on alumina substrate ($\epsilon_r = 10$). (a) $f = 1$ GHz. (b) $f = 2$ GHz. (c) $f = 4$ GHz. (d) $f = 8$ GHz.

which is similar to [5, eq. (25)] except for a correction factor $2Q_0/n$. In terms of measured results, P_r/P_t is given by

$$\frac{P_r}{P_t} = \frac{Q_0' - Q_t'}{Q_0'} \tag{18}$$

where Q_0' is the measured Q of the microstrip resonator when located in a waveguide below cutoff, and Q_t' is the value obtained without shielding. A critical coupling technique [1], [9] was used to measure the Q's of the transmission-line resonator.

A comparison between the theoretical and experimental fractional radiated power is given in Tables I and II. Table I is for resonators with a fixed dielectric constant and varying h/λ, while Table II is for resonators having approximately the same h/λ but varying ϵ_r.

IV. CONCLUSION

Formulas have been derived for the radiation Q of open-circuited microstrip resonators based on the original work of Lewin. These relations were used to prepare a series of design graphs illustrating the effect of radiation on the overall circuit Q as a function of characteristic impedance, frequency, dielectric constant, and substrate thickness. The results show clearly the dominating influence of radiation losses at higher frequencies, especially for low-impedance lines on substates with a low dielectric constant. Good agreement between theoretical and experimental results was obtained with multiple $\lambda_g/2$ resonators over a large range of normalized substrate thicknesses, h/λ and dielectric constants.

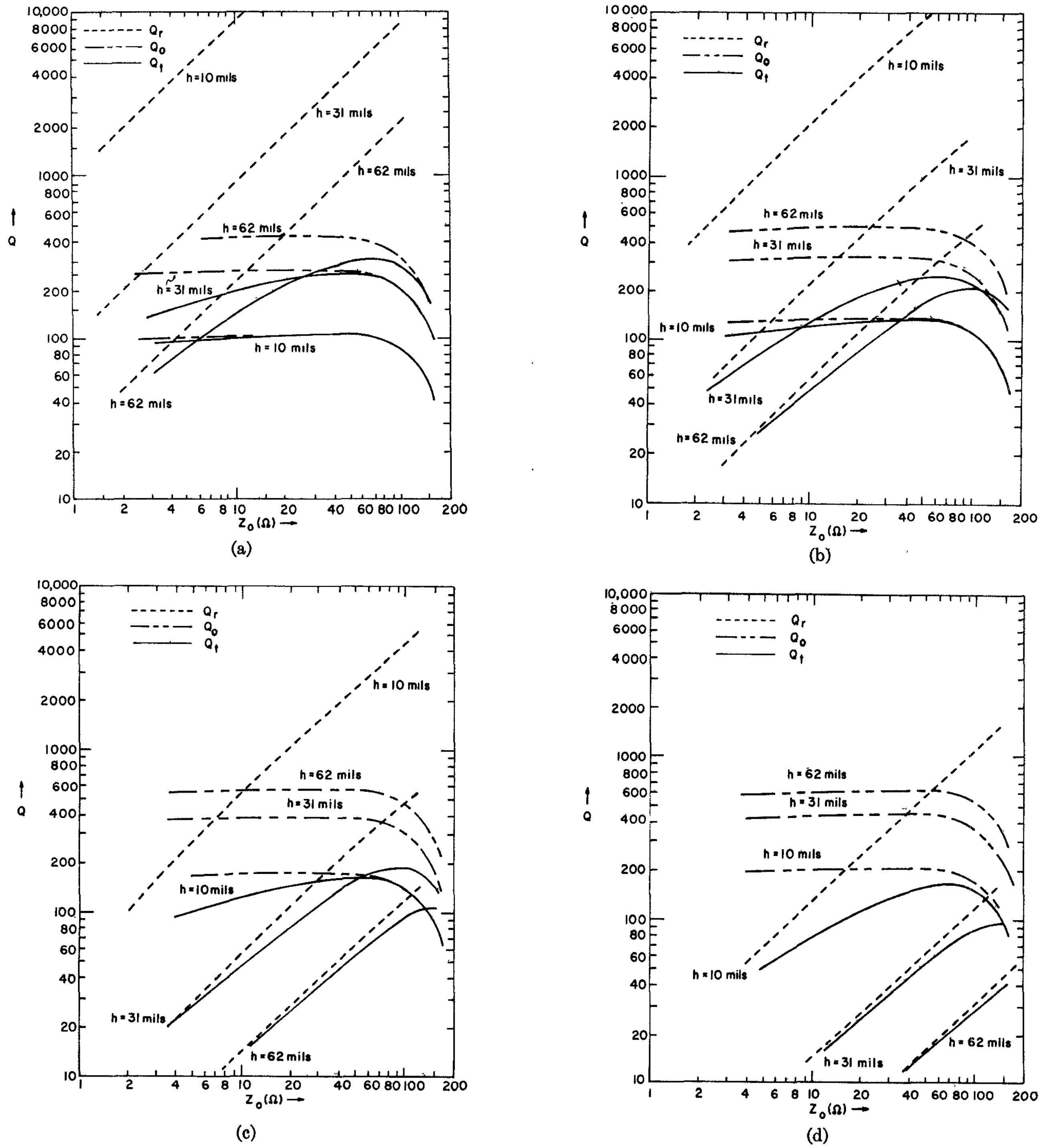

Fig. 3. Q factors for $\lambda g/4$ microstrip resonator on Duroid substrate ($\epsilon_r = 2.35$). (a) $f = 1$ GHz. (b) $f = 2$ GHz. (c) $f = 4$ GHz. (d) $f = 8$ GHz.

TABLE I

FRACTIONAL AMOUNT OF RADIATED POWER (P_r/P_t) VERSUS NORMALIZED SUBSTRATE THICKNESS (h/λ) WITH $\epsilon_r = 2.47$

h/λ	Mode No. n	Q_o'	Q_t'	$(P_r/P_t)_{exp.}$ (eq'n 15)	$(P_r/P_t)_{theor.}$ (eq'ns 13-14)
.00537	2	340	300	0.120	0.136
.00609	2	345	298	0.136	0.171
.00807	3	360	291	0.190	0.201
.00931	3	370	286	0.227	0.257
.01069	2	540	366	0.320	0.499
.01603	3	605	194	0.680	0.626

TABLE II

FRACTIONAL AMOUNT OF RADIATED POWER (P_r/P_t) VERSUS SUBSTRATE DIELECTRIC CONSTANT ϵ_r WITH $h/\lambda = 0.009$

ϵ_r	ϵ_{eff}	n	Q_o'	Q_t'	$(P_r/P_t)_{exp}$ (eq'n 15)	$(P_r/P_t)_{theor.}$ (eq'ns 13-14)
2.47	2.25	3	370	286	0.227	0.257
6.0	4.3	2	213	183	0.140	0.133
9.0	6.0	3	273	263	0.040	0.080
16.0	10.0	2	152	148	0.030	0.020

REFERENCES

[1] R. A. Pucel, D. J. Masse, and C P. Hartwig, "Losses in microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 342–350, June 1968.
——, "Correction to 'Losses in microstrip,'" *ibid.* (Corresp.), vol. MTT-16, p. 1064, Dec. 1968.

[2] E. J. Denlinger, "Radiation from microstrip resonators," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-17, pp. 235–236, Apr. 1969.

[3] H. Sobol, "Radiation conductance of open-circuit microstrip," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 885–887, Nov. 1971.

[4] ——, "A review of the technological and electromagnetic limitations of hybrid circuits for microwave applications," *IEEE Trans. Parts Hybrids, Packag.*, vol. PHP-8, pp. 59–66, June 1972.

[5] G. Vendelin, "Limitations on stripline Q," *Microwave J.*, pp. 63–69, May 1970.

[6] L. Lewin, "Radiation from discontinuities in strip line," Inst. Elec. Eng. Monograph #1358E, pp. 163–170, Feb. 1960.

[7] S. P. Morgan, "Effect of surface roughness on eddy current losses at microwave frequencies," *J. Appl. Phys.*, vol. 20, pp. 352–362, Apr. 1949.

[8] H. J. Carlin, "A simplified circuit model for microstrip," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 589–591, Sept. 1973.

[9] E. L. Ginzton, *Microwave Measurements.* New York: McGraw-Hill, 1957, pp. 391–434.

Microwave Measurement of the Temperature Coefficient of Permittivity for Sapphire and Alumina

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Abstract—**Measurements of the temperature coefficients of permittivity and of thermal expansion, for the important MIC substrate materials alumina and sapphire, are reported. The results are presented and in the case of sapphire include figures for the two main crystal orientations. An interesting correlation exists between our results for alumina substrates and those for sapphire substrates in which the optical axis is perpendicular to the plane of the slice. The temperature stability of resonators on sapphire and alumina is discussed and experimental data are presented.**

INTRODUCTION

Knowledge of the temperature dependence of the properties of microwave integrated circuit (MIC) substrate materials can be as important as precise knowledge of the properties themselves. Recently, we presented [1] a technique for precise measurement of substrate permittivity. In this short paper we describe the measurement of the temperature coefficient of permittivity for sapphire and alumina substrates by a refinement of the earlier method.

The substrates used for this work were 25 mm square by 0.5 mm thick and they were metallized all over to form resonant cavities. For each resonant mode the resonance frequency and the unloaded cavity Q factor were measured as functions of cavity temperature. These data, in conjunction with separate measurement of the substrate expansion coefficients, were used to determine the temperature coefficients of permittivity.

Fig. 1 illustrates one of our cavities which is formed by first metallizing the substrate on all sides and then etching a single slot on one edge for coupling. A standard launcher is brought into contact with the cavity so that the flat tab lies on the broad-face metallization and above the center of the coupling slot. The cavity is held

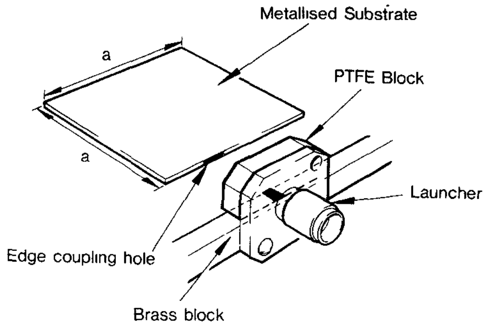

Fig. 1. Metallized-substrate cavity and launcher.

firmly between the launcher tab and a brass block which is screwed to the launcher flange. The $\boldsymbol{E}$ and $\boldsymbol{H}$ fields in the coaxial feed line approximately match the corresponding cavity fields at the slot, and coupling will be by $\boldsymbol{E}$ and/or $\boldsymbol{H}$ field depending on the mode excited. The coupling coefficient is mode dependent.

THEORY

If the substrate medium is isotropic then TE modes may be excited in the cavity of Fig. 1 with the $\boldsymbol{E}$ field directed across the thin dimension. The substrate dielectric constant ϵ, substrate dimension a, and mode frequencies $f_{n,m}$ in the range from 2 to 12 GHz are related, to a high degree of accuracy, by (1) in which n and m are integers and c is the velocity of light in vacua

$$\epsilon(f,T) = \frac{c^2}{a^2 f^2_{n,m}} \left\{\frac{m^2+n^2}{4}\right\}. \quad (1)$$

If the substrate medium is sapphire the excited modes will still be TE provided one of the major crystal axes is directed across the thin dimension of the slice. The permittivity tensor for these orientations reduces to a diagonal matrix and (1) remains valid; but the permittivity value given by the equation is that parallel to the $\boldsymbol{E}$ field i.e., the permittivity across the thin dimension.

In deriving (1) cavity losses and other second-order effects have been neglected. The frequency-pulling effect of the temperature-dependent cavity losses may be taken into account by considering the resonance frequency f as a function of both the unloaded Q factor, Q_0, and the cavity temperature T, i.e.,

$$f_{n,m} = f(Q_0,T). \quad (2)$$

Differentiating (1) with respect to T and using (2) we may derive the partial differential equation (3) i.e.,

$$\frac{1}{\epsilon}\frac{\delta\epsilon}{\delta T} = -\alpha_1 - \alpha_2 - \frac{1}{f}\frac{df}{dT}\left\{2 + \frac{f}{\epsilon}\frac{\delta\epsilon}{\delta f} - \frac{2(\delta f/\delta Q_0)(dQ_0/dT)}{df/dT}\right\}. \quad (3)$$

The constants α_1 and α_2 are the thermal expansion coefficients $(1/a)$ (da/dT) for the sides "a" in Fig. 1, which may be different for anisotropic materials, and in separating these in (3) we have assumed that they are not greatly different. The $\delta\epsilon/\delta f$ term may be neglected provided that it is small compared with the last (Q_0) term. A straightforward comparison of the two terms using representative experimental data yields this condition as

$$\frac{1}{\epsilon}\frac{\delta\epsilon}{\delta f} \ll 0.01/\text{GHz}.$$

Although we have been unable to find a published figure over our range of temperatures and frequencies, [2] implies there is no variation at all in the frequency coefficient below 400°C. In the absence of any evidence to the contrary, we therefore feel justified in neglecting it. The factor $\delta f/\delta Q_0$ is calculable [3] and is given by

$$\frac{\delta f}{\delta Q_0} = \frac{f}{2Q_0{}^2}. \quad (4)$$

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